idea of the range of applicability of Cohen's solution, the percentage error in J_+ [from his Eq. (19)] as a function of r_p^* for constant values of K is given in Fig. 8.

Cicerone and Bowhill obtain numerical solutions for the attracted ion distribution at large probe potential and the corresponding probe characteristics. The characteristics presented, covering a range of $.1 < \rho_p < 1$ and an approximate potential range of $20 < \Phi_p^* < 500$, agree with the current results in the overlapping potential range, at least to within the accuracy attainable in reading the plotted data.

Conclusions

Numerical solutions of the spherical continuum electrostatic probe equations have been obtained for the case of equal electron and ion temperatures for a wide range of probe radius to Debye length ratio. Since the exact numerical solutions of the complete equations are straightforward and economical to obtain (in the range 15 to 25 seconds per solution on a CDC 6500 computer), it appears that further investigations of these equations should be aimed at finding simplified solutions of closed form. The numerical results presented here should prove valuable in assessing the accuracy of any such solutions.

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Thin-Wire Langmuir-Probe Measurements in the Transition and Free-Molecular Flow Regimes

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Thin-wire Langmuir probes aligned with the flow direction have been used to measure the electron temperature and electron density in the inviscid nozzle flow of a short-duration reflected-shock tunnel. The electron density was inferred from the ion current portion of the probe characteristic and was simultaneously measured using microwave interferometers. The test gas used in these experiments was nitrogen at an equilibrium reservoir condition of 7200°K and 17.1 atm. At selected nozzle locations, the probe diameter and fineness ratio (L/D) of the probe were systematically varied in order to investigate probe performance in the transition and free-molecular flow regimes. The measured electron temperatures did not depend upon the probe diameter or the fineness ratio. The electron density inferred from the probe characteristic was found to be sensitive to collisional effects but insensitive to the fineness ratio in both the transition and free-molecular flow regimes. For free-molecular flow the results agree with Laframboise's theoretical predictions for ion collection in a portion of the orbital-motion-limited region. For probes having a radius less than a debye length in a free-molecular flow, the experimental results appear to disagree with the theory, the collected currents being larger than those predicted. In the transition-flow regime, the correction for collisional effects given by Talbot and Chou provides good agreement between the Langmuir-probe and microwave-interferometer electron-density data.

I. Introduction

THE behavior of thin-wire Langmuir probes in hypersonic flows is of interest because these probes provide a means for obtaining local measurements of electron temperature and electron density. Several authors[1–7] have demonstrated the utility of cylindrical probes in flow environments that are relatively well understood. However, even for such flow situations, there are still many aspects of electrostatic-probe operation that are not understood. Relatively few experiments have been reported in the literature that have systematically investigated the influence of various plasma and probe parameters on the electron density and electron temperatures deduced from the current-voltage characteristic. It is therefore the purpose of this paper to report the results of an experimental study that was undertaken in an effort to improve the understanding of thin-wire probes in hypersonic flows.

Several authors[8–12] have presented theoretical results for the current collected by thin-wire probes. Laframboise[8] has developed a numerical scheme for obtaining an iterative solution of the Bernstein and Rabinowitz[10] formulation. He presented tabulated results for the collected current for a wide range of probe potentials, ratios of probe radius to debye length, and ratios of ion temperature to electron temperature. These results have been used to interpret most of the

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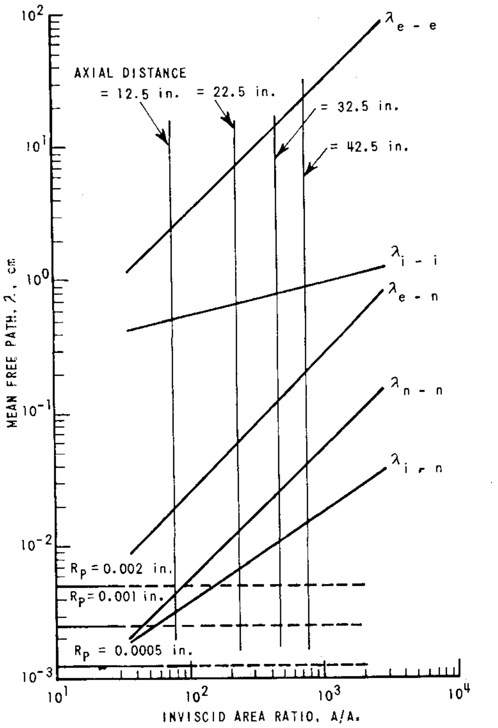

Fig. 1 Calculated mean free paths in expanding nitrogen flow.

probe data obtained in the present work. Analyses of the current collection by spherical probes in the transition regime have been presented by Bienkowski and Chang,[13] by Chou, Talbot and Willis[14], and by Wasserstrom, Su, and Probstein.[15] Talbot and Chou[16] have developed an approximate solution which is simple to use and is applicable to cylindrical probes.

The early experiments using thin-wire probes in flowing environments were conducted in free jets.[1-4] More recently, data have been obtained from these probes in shock-tunnel flows.[5-7] In this paper data are presented for both the transition-flow and free-molecular flow regimes of probe operation. In Ref. 5 it was found that increasing the L/D from 50 to 200 resulted in about a 40% decrease in collected current density. The results reported in this paper are in agreement with those of Refs. 1 and 2 in that they indicate little or no effect of L/D. The applicability of Laframboise's theoretical prediction in at least a portion of the orbital-motion-limited region of probe operation has been verified on the basis of data presented here. This result disagrees with the findings reported in Refs. 3 and 5. However, neither of these previous investigators simultaneously obtained an independent measurement of electron density in order to arrive at their conclusion. Such an independent measurement was obtained in this work using microwave interferometry. Finally, the electron tem perature was found to be independent of flow regime and fineness ratio.

2. Experimental Apparatus and Technique

A pressure-driven shock tube was used to produce a reservoir of high-temperature nitrogen which was subsequently expanded in a conical nozzle constructed of Fiberglas. The test gas used in these experiments was UPC nitrogen supplied by Air Products and Chemicals Inc. A chemical analysis of the gas indicated the following: oxygen less than 0.5 ppm, total hydrocarbons less than 1 ppm, and water less than 0.15 ppm. The shock tube was purged with the test gas to approximately 5 torr just prior to each run.

Flow conditions were monitored by simultaneous measurement of near-infrared radiation intensity and the reflected-shock pressure in the shock tube and visible radiation-intensity at several axial locations in the nozzle. A detailed discussion of the shock tube and nozzle measurements is given in Refs. 17 and 18.

The electron density and electron temperature were measured on the nozzle centerline at axial stations of 12.5, 22.5, 32.5, and 42.5 in. from the throat using thin-wire Langmuir probes aligned with the flow direction. The ion current portion of the probe characteristic was used to infer the electron density. An independent and simultaneous measurement of electron density was obtained one inch upstream of each of the probe stations using microwave interferometers operating at frequencies of either 35 or 17 GHz.

The probes were constructed by surrounding tungsten wires with a quartz envelope, leaving a specified length of bare wire exposed. The wire diameters used in these experiments were 0.004, 0.002, and 0.001 in. For the 0.002-in.-diam wire, three values of the fineness ratio (50, 100, and 200) were used. The fineness ratio for the 0.004- and 0.001-in.-diam probes was 100. Immediately prior to each run the probe was ultrasonically cleaned in a dilute solution of sodium hydroxide to remove the tungsten oxide. Each probe was used for a single run and then discarded.

The voltage applied to the probe was swept from -6 to $+1$ v (relative to ground) in approximately 80 μsec, which is slow enough to avoid transient effects.[19] The experimental procedure used to obtain the Langmuir-probe data has been described in detail in a previous paper.[20]

3. Discussion of Results

The influence of flow regime (transition or free-molecule) and fineness ratio on the electron density and electron temperature deduced from the current voltage characteristic of a thin-wire Langmuir probe has been determined for an expanding nitrogen plasma. The test gas was expanded from an equilibrium reservoir condition of 7200°K and 17.1 atm. Typical Langmuir-probe and microwave-interferometer data records obtained in this plasma are presented in Ref. 21 and the data reduction procedure was discussed in Refs. 7 and 21. Also included in Ref. 21 is a discussion of the neutral and charged species distributions in the nozzle flow. The dominant ion at the measuring stations was found to be N^+ and thus the probe data were reduced using this value of the ion mass in the theories of Refs. 8, 16, and 22. The calculated heavy-particle gas velocity varied nearly linearly from approximately 14,400 fps at the 11.5-in. station to approximately 15,200 fps at the 41.5-in. measuring station.

Figure 1 presents the relative magnitudes of the mean free paths λ_{i-n}, λ_{n-n}, λ_{e-n}, λ_{i-i}, and λ_{e-e} as a function of inviscid area ratio. These values were calculated using the expressions summarized by Sonin[2] and the nozzle-flow properties calculated in Ref. 21. For the purpose of estimating mean free paths, the electron temperature can be assumed to be equal to the heavy-particle translational temperature from the reservoir value down to 3500°K at which point it can be considered to freeze (see Fig. 2). The Langmuir-probe measuring stations of 12.5, 22.5, 32.5, and 42.5 in. are located on Fig. 1. Also included on Fig. 1 is the relative magnitude of the various probe radii used, i.e., 0.002, 0.001,

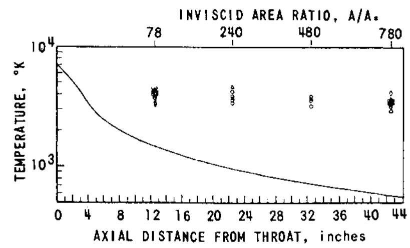

Fig. 2 Measured electron temperature in expanding nitrogen flow.

and 0.0005 in. At all of the measuring stations the electron mean free paths are large compared to the probe radius. Because of the relative insensitivity of electron collection to the magnitude of the ion-neutral and neutral-neutral mean free paths, it is anticipated that the electron temperature deduced from the electron retarding region, where the electron current is much larger than the ion current, should be correct. Figure 1 further shows that at the 12.5-in. measuring station the 0.004-in. and 0.002-in.-diam probes should not provide the correct electron densities when interpreted in terms of Laframboise's theory because current collection should be inhibited by the relatively small values of λ_{i-n} and λ_{n-n}. However, the 0.001-in.-diam probe should provide electron densities in reasonable agreement with the microwave-interferometer value. At the 22.5-in. measuring station the current collected by the 0.004-in.-diam probe should be inhibited but that collected by the 0.002-in.-diam probe should not. Beyond 22.5 in. from the throat collision effects should not influence the electron densities deduced from the current-voltage characteristic of the probe.

3.1 Influence of Flow Regime and Fineness Ratio on Electron Temperature Deduced from Probe Characteristic

Electron temperatures measured at several axial locations in an expanding nitrogen plasma using probe diameters of 0.004, 0.002, and 0.001 in. and fineness ratios of 200, 100, and 50 are compared in Fig. 2 (the key to the experimental data for Figs. 2, 3, and 4 is given in Table 1). The calculated heavy-particle translational temperature (solid line) is also included for comparison purposes. The measurements obtained at the 12.5-in. location using the 0.004-in. and 0.002-in.-diam probes are in transition flow with respect to the ion-neutral and neutral-neutral mean free paths while the 0.001-in. results are in free-molecule flow. However, within the scatter of the experimental data, the electron-temperature measurements are all in good agreement confirming the assertion that electron collection is independent of the ion mean free paths. The same result is illustrated at the 22.5-in. location for 0.004-in. (transition-flow) and 0.002-in.-diam (free-molecular flow) probes. The diameter of the probe was then held fixed at 0.002 in. while the fineness ratio was varied from 50 to 200 and the measurements were repeated at the 12.5- and 42.5-in. locations. Within the accuracy of the experimental data, it was not possible to observe any influence of L/D on the deduced electron temperature. This conclusion is consistent with the findings of Sonin[2] for a free-molecule flow environment.

3.2 Influence of Flow Regime and Fineness Ratio on Electron Density Deduced from Probe Characteristic

The electron-densities deduced from the current-voltage characteristic of the probe using Laframboise's theoretical results are compared in Fig. 3 with those measured using the microwave interferometers. Consider first the measurements obtained at the 12.5-in. location for a constant value of L/D equal to 100 but probe diameters of 0.004, 0.002, and 0.001 in.

Table 1 Key to Langmuir-probe and microwave interferometer data given on Figs. 2–4

KEY	PROBE DIAMETER, D in.	PROBE LENGTH, L in.	L/D
○	0.004	0.400	100
△	0.002	0.200	100
◇	0.002	0.100	50
▭	0.002	0.400	200
⬭	0.001	0.100	100
□	MICROWAVE-INTERFEROMETER DATA		

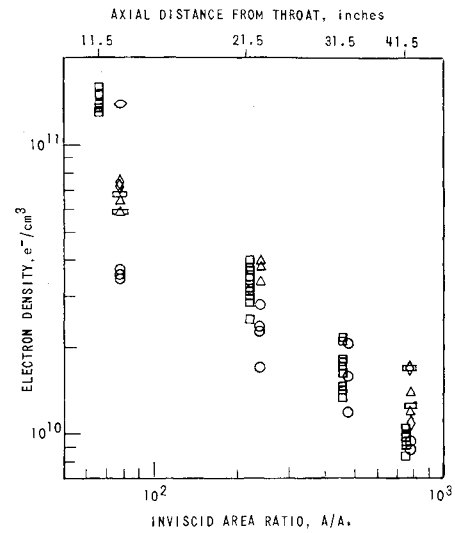

Fig. 3 Measured electron densities in expanding nitrogen plasma.

The electron-densities obtained with the 0.004-in. probes were approximately 25% of the microwave-interferometer values whereas those obtained with the 0.002-in. probes were approximately 60% of the interferometer values. By decreasing the probe diameter to 0.001 in. so that the free-molecular flow theoretical results[8] should be directly applicable, the electron densities deduced from the Langmuir probe were found to be very nearly equal to those measured with the microwave-interferometer. The 0.002- and 0.004-in.-diam probes were also used at the 22.5-in. location. The electron densities obtained with the larger diameter probe were generally below those measured with the microwave interferometers. However, the results obtained with the smaller diameter probe were in good agreement with the microwave data as shown on Fig. 3.

The influence of fineness ratio on the electron density deduced from a thin-wire probe current-voltage characteristic was investigated at the 12.5- and 42.5-in. locations. The

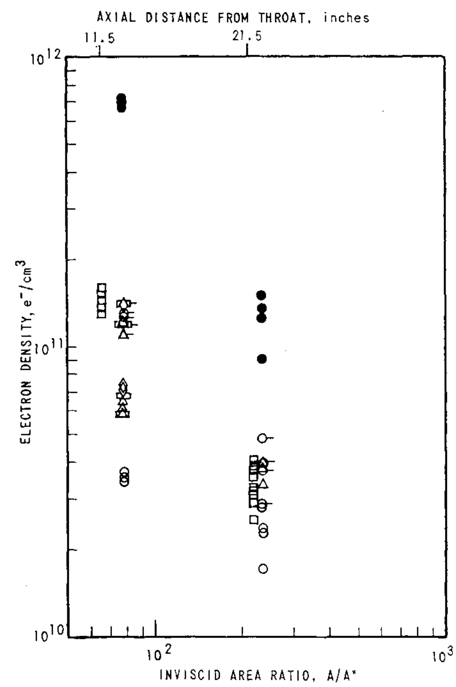

Fig. 4 Comparison of experimental data with predictions of free-molecular flow, transition flow, and continuum-flow theories.

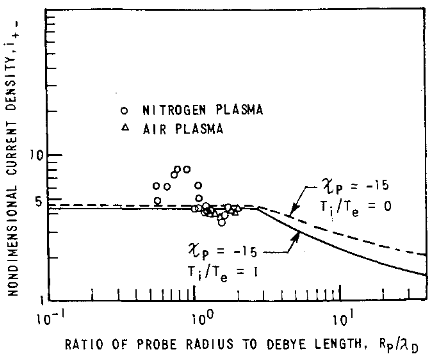

Fig. 5 Comparison of experimental data with theory.

Table 2 Comparison of experimental data with free-molecular flow and transition-flow predictions

Probe location, in.	Probe diameter, in.	Probe L/D	e^- from μ-waves just upstream of probe, e^-/cm^3	e^- from Laframboise, e^-/cm^3	e^- from Talbot & Chou, e^-/cm^3
12.5	0.004	100	1.3×10^{11}	3.6×10^{10}	1.3×10^{11}
12.5	0.004	100	1.4×10^{11}	3.4×10^{10}	1.2×10^{11}
12.5	0.004	100	1.3×10^{11}	3.7×10^{10}	1.3×10^{11}
12.5	0.002	100	1.2×10^{11}	6.4×10^{10}	1.2×10^{11}
12.5	0.002	100	1.5×10^{11}	5.8×10^{10}	1.1×10^{11}
12.5	0.002	200	1.2×10^{11}	6.0×10^{10}	1.2×10^{11}
12.5	0.002	200	1.3×10^{11}	6.8×10^{10}	1.4×10^{11}
12.5	0.002	50	1.4×10^{11}	7.4×10^{10}	1.4×10^{11}
12.5	0.002	50	...	7.2×10^{10}	1.4×10^{11}
22.5	0.004	100	3.7×10^{10}	2.2×10^{10}	3.8×10^{10}
22.5	0.004	100	4.0×10^{10}	2.8×10^{10}	4.8×10^{10}
22.5	0.004	100	3.1×10^{10}	2.3×10^{10}	3.9×10^{10}
22.5	0.004	100	3.2×10^{10}	1.7×10^{10}	2.8×10^{10}

probe diameter was held fixed at 0.002 in. but three values of fineness ratio were used: 50, 100, and 200. Therefore, the probe measurements obtained at the upstream station are in transition-flow whereas those obtained downstream are in free-molecular flow. The results illustrated that the influence of the fineness ratio on the deduced electron density was within the scatter of the experimental data for both of these flow regimes. Both Clayden[1] and Sonin[2] previously arrived at this conclusion (Clayden found he could use an L/D as low as 34 with no effect) for a free-molecule-flow environment. This conclusion is in disagreement with that of Lederman, Bloom, and Widhopf,[5] who found, for a free-molecule-flow situation, that the collected current density decreased by approximately 40% when the fineness ratio was increased from 50 to 200.

The electron-density data collected at the 12.5-in. location using the 0.004 and 0.002-in. diam probes and that collected at the 22.5-in. location using the 0.004-in.-diam probes could, in principle, be interpreted using existing theoretical treatments[13–15] for the transition-flow regime, adapted to the cylindrical-probe case. Application of these theories to the experimental results reported here is beyond the scope of this paper. However, a simplified analysis has been provided by Talbot and Chou[16] that was easily applied to the transition-regime data. The analysis of Ref. 16 is basically a scheme for interpolating between the free-molecular results of Laframboise[8] and the collision dominated results of Shulz and Brown.[23] (In our calculations, the actual values of i_{+-} tabulated by Laframboise were used because Talbot and Chou's fit is not sufficiently accurate for the orbital-motion-limited region.)

The upstream electron-density data are re-plotted on Fig. 4 and are compared with the number densities inferred from the probe data using the collisionless theory[8] (open symbols), the transition correlation[16] (flagged symbols), and the continuum theory[22] (closed symbols). With the exception of the latter values, these data are also listed in Table 2. When the continuum theory is used the inferred number densities are much greater than those given by the microwave data. However, by applying the interpolation scheme‡ of Ref. 16, relatively good agreement was achieved between the microwave-interferometer data and the probe data as shown on Fig. 4 and in Table 2.

At the 42.5-in. location, the electron densities obtained with the 0.002-in.-diam probes were greater than those measured with the 0.004-in. probes and the microwave interferometers. The value of R_P/λ_D was less than 1.0 for these smaller probes at this measuring station. The results indicate that it may be necessary to modify the value of the theoretical nondimensional current when R_P/λ_D becomes less than 1. The next section discusses these results in more detail.

3.3 Comparison of Experimental Results with Laframboise's Theoretical Results in Orbital-Motion-Limited Region

In Fig. 5 the experimental data reported in this paper and in Ref. 20 are compared to theoretical values[8] by plotting nondimensional current density, i_{+-}, where $i_{+-} = j_i/en_e(kT_e/2\pi m_i)^{1/2}$ vs the ratio of probe radius to debye length, R_P/λ_D. Only those data obtained under conditions for which the probe was considered to be in a free-molecule flow environment are included on this plot. The experimental data span a range of R_P/λ_D from approximately 0.5–2.0. The portion of the predicted distribution for which i_{+-} is constant corresponds to the orbital-motion-limited values of the ion current. In this region there is no potential barrier to the collection of ions and thus they are prohibited from reaching the probe surface only by their orbital motion around the probe.

Both the previous air[20] and the current nitrogen data are in good agreement with the prediction for $1 \leq R_P/\lambda_D \leq 2$ thus providing confidence in the existence of such a region. For smaller values of R_P/λ_D the experimental data suggested values of i_{+-} that were generally greater than the predicted i_{+-}. For values of $R_P/\lambda_D < 1$ the ion sheath will be quite thick. Consequently, the current collection§ by a thin-wire probe may be governed by the rate of convection of the ions to the sheath edge, as suggested by deBoer, Johnson, and Thompson.[26]

The results reported in Fig. 5 are qualitatively consistent with those reported by Sonin[3] and Lederman, Bloom, and Widhopf.[5] However, neither of these references reported a region of probe operation that corresponded to the orbital-motion-limited region of Laframboise's theory. In addition, for the same values of R_P/λ_D, these authors found significantly greater values of i_{+-} necessary to correlate their data.

4. Conclusions

Thin-wire Langmuir probes and microwave interferometers have been used to investigate the influence of flow regime and fineness ratio on the electron temperatures and electron densities deduced from the probe current-voltage characteristic. The experiments were performed in the inviscid nozzle flow of a reflected-shock hypersonic shock tunnel. Nitrogen was used as the test gas and was expanded from an equilibrium reservoir condition of 7200°K and 17.1 atm.

‡ In the data reduction the value $\mu = 3.3\ \rho_0/\rho$ cm^2/v sec was used for the mobility of the N^+ ion, where ρ_0 is the gas density at 0°C and 1 atm pressure and ρ is the local gas density. McDaniel[24] gives the low-field value of μ at standard conditions as 3.3 cm^2/v sec. The dependence on density was assumed on the basis of the formula for μ given by Loeb.[25]

§ After this paper was submitted for publication, Hester and Sonin[27] published a report in which they explain all of the previous data for $R_P/\lambda_D \leq 1$ in terms of an end effect caused by convection.

The electron temperatures measured with the probes were found to be independent of both flow regime and fineness ratio. However, the electron densities (deduced using free-molecular theory[8]), were found to deviate from the microwave interferometer data unless the free-molecule flow restriction was met. If the flow was free-molecular and if $R_P/\lambda_D \geq 1$ then the probe data were found to be in good agreement with the interferometer data. For $R_P/\lambda_D < 1$, the effects of convection appear to become important in hypersonic flows. The agreement between the probe and microwave-interferometer data for $1 \leq R_P/\lambda_D \leq 2$ confirms the validity of Laframboise's[8] theoretical prediction for ion collection in this portion of the orbital-motion-limited region.

The influence of fineness ratio, L/D, on the electron densities determined from the probe data was found to be insignificant for both the transition and free-molecule flow regimes for the range of L/D used here (50, 100, 200). The current collected in the transition-flow regime appears to be adequately predicted by the analysis of Talbot and Chou.

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